

THE COLLEGE FINALS



The Finals will be conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

The last person remaining wins an additional \$75 and will be crowned the **Integration Champion!**

INTEGRAL #1

**READY,
GET SET,...**

2:30



INTEGRAL #1

$$\int \frac{\ln x}{x^2} dx$$

$$\left[\begin{array}{l} \text{integrate by parts:} \\ u = \ln x \\ du = \frac{1}{x} dx \end{array} ; \quad \begin{array}{l} dv = \frac{1}{x^2} dx \\ v = -\frac{1}{x} \end{array} \right]$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$= -\frac{1 + \ln x}{x} + C$$

INTEGRAL #2

**READY,
GET SET,...**

2:30

INTEGRAL #2

$$\int (e^x + e^{-x})^2 dx$$

INTEGRAL #2

$$\int (e^x + e^{-x} x x)$$

INTEGRAL #3

**READY,
GET SET,...**

2:30

INTEGRAL #3

$$\int e^{2x} \sqrt{1 + e^{2x}} dx$$

INTEGRAL #3

$$\int e^{2x} \sqrt{1 + e^{2x}} dx$$

$$= \frac{1}{2} \int \sqrt{u} du \quad [u = 1 + e^{2x}; \quad du = 2e^{2x} dx]$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \frac{(1 + e^{2x})^{3/2}}{3/2} + C$$

INTEGRAL #4

**READY,
GET SET,...**

2:30



INTEGRAL #4

$$\int \frac{\ln \sqrt{x}}{x} dx$$

$$= \int \frac{\ln x^{1/2}}{x} dx = \frac{1}{2} \int \frac{\ln x}{x} dx$$

$$= \frac{1}{2} \int u du \quad \left[u = \ln x; \quad du = \frac{1}{x} dx \right]$$

$$= \frac{u^2}{4} = \frac{(\ln x)^2}{4} + C$$

INTEGRAL #5

**READY,
GET SET,...**

2:30

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$$

INTEGRAL #5

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx$$

$$= \frac{1}{2} \int \frac{1}{u} du \quad [u = e^{2x} + e^{-2x}; \quad du = (2e^{2x} - 2e^{-2x}) dx]$$

$$= \frac{1}{2} \ln u + C$$

$$= \frac{\ln(e^{2x} + e^{-2x})}{2} + C$$

INTEGRAL #6

**READY,
GET SET,...**

2:30

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

INTEGRAL #6

$$\int x \sec^2 x \, dx$$

$$\left[\begin{array}{l} \text{integration by parts:} \\ u = x \quad ; \quad dv = \sec^2 x \, dx \\ du = dx \quad ; \quad v = \tan x \end{array} \right]$$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x + \ln|\cos x| + C$$

INTEGRAL #7

**READY,
GET SET,...**

2:30

INTEGRAL #7

$$\int x \left(1 + \frac{1}{x} \right)^3 dx$$

INTE

$$\int x \left(1 + \frac{1}{x} \right)^3 dx$$

$$= \int x \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3} \right) dx$$

$$= \int \left(x + 3 + \frac{3}{x} + \frac{1}{x^2} \right) dx$$

$$= \frac{x^2}{2} + 3x + 3 \ln x - \frac{1}{x} + C$$

INTEGRAL #8

**READY,
GET SET,...**

2:30

INTEGRAL #8

$$\int \cos^5 x \, dx$$

INTEGRAL #8

$$\int \cos^5 x \, dx$$

$$= \int (\cos^2 x)^2 \cos x \, dx = \int (1 - \sin^2 x)^2 \cos x \, dx$$

$$= \int (1 - u^2)^2 \, du \quad [u = \sin x; \quad du = \cos x \, dx]$$

$$= \int (u^4 - 2u^2 + 1) \, du = \frac{\sin^5 x}{5} - \frac{2 \sin^3 x}{3} + \sin x + C$$

INTEGRAL #9

**READY,
GET SET,...**

2:30

INTEGRAL #9

$$\int \frac{x^2 + 1}{(x + 1)^2} dx$$

INTEGRAL #9

$$\int \frac{x^2 + 1}{(x + 1)^2} dx$$



INTEGRAL #11

**READY,
GET SET,...**

2:30

INTEGRAL #11

$$\int \frac{1}{x^2 - 1} dx$$



$$\int \frac{1}{x^2 - 1} dx$$

$$\left[x = \sec \theta ; \quad dx = \sec \theta \tan \theta d\theta ; \quad \sqrt{x^2 - 1} = \tan \theta \right]$$

$$= \int \frac{\sec \theta \tan \theta}{\tan \theta} d\theta = \int \sec \theta d\theta$$

$$= \ln|\sec \theta + \tan \theta| + C = \ln|x + \sqrt{x^2 - 1}| + C$$

INTEGRAL #12

**READY,
GET SET,...**

2:30

INTEGRAL #12

$$\int \sin^2 x \cos^2 x \, dx$$



$$\int \sin$$