

# INTEGRATION BEE FINALS

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The finals are conducted in rounds. One at a time, each remaining contestant will have **two and a half minutes** to compute an indefinite integral. If answered correctly, the contestant remains in the competition. Once every remaining contestant has attempted one problem, a round is completed. If during any round, all contestants are unable to complete a problem correctly, all contestants will remain in the competition for another round.

**Contestants must circle their final answer.** Contestants do **not** need to include the constant of integration  $+ C$  in their answer.

**The last person remaining wins an additional \$75 and will be crowned the Integration Champion!**

READY,  
GET SET,...

2:30

INTEGRAL # 1

Z

$$\frac{x^2 + x^6}{1 + x^2} dx$$

## INTEGRAL # 1

$$\int \frac{x^2 - x^6}{1 - x^2} dx$$
$$= \int \frac{x^2(1 - x^2)(1 + x^2)}{1 - x^2} dx$$

$$= \int (x^2 + x^4) dx$$

$$= \frac{x^3}{3} + \frac{x^5}{5} + C$$

READY,  
GET SET,...

2:30

INTEGRAL # 2

Z

$$\frac{1 + \frac{x}{x+1}}{\quad}$$



READY,  
GET SET,...

2:30



Z

$$\int \frac{x}{x^4 + 2x^2 + 1} dx$$

## INTEGRAL # 3

Z

$$\frac{x}{x^4 + 2x^2 + 1} dx$$

Z

$$= \frac{x}{(x^2 + 1)^2} dx$$

Z

$$= \frac{1}{2} \frac{1}{u^2} du \quad u = x^2 + 1, \quad du = 2x dx$$

$$= \frac{1}{2u} + C$$

$$= \frac{1}{2(x^2 + 1)} + C$$

READY,  
GET SET,...

2:30



## INTEGRAL # 4

$$\int \frac{x^2 + x + 1}{(2x + 1)^2} dx$$

$$= \int \frac{x^2 + x + 1}{4x^2 + 4x + 4} dx$$

$$= \int \frac{1}{4} dx$$

$$= \frac{1}{4}$$

READY,  
GET SET,...

2:30

## INTEGRAL # 5

Z

$$\int \frac{1}{x+7} + \frac{1}{x} dx$$

INTEGRAL # 5

Z

$$\int \frac{1}{x+7} + \frac{1}{x} dx$$

$$= \int \frac{1}{x+7} + \frac{1}{x} \cdot \frac{x}{x} + \frac{1}{x+7} \cdot \frac{x}{x} dx$$

$$= \int \frac{x}{x+7} + \frac{x}{x} dx$$

$$= \frac{2}{21} (x+7)^{3-2} x^{3-2} + C$$



READY,  
GET SET,...

2:30

## INTEGRAL # 6

$$\int \frac{1}{4 + x^2} dx$$

INTEGRAL # 6

$$\int \frac{1}{4 + x^2} dx$$

$$= \int \frac{1}{2 + \frac{1}{x^2}} dx$$

$$= \int \frac{1}{x^2 + \frac{1}{x^2}} dx$$

$$= \int \left( x^2 + \frac{1}{x^2} \right) dx = \frac{x^3}{3} - \frac{1}{x} + C$$

READY,  
GET SET,...

2:30



## INTEGRAL # 7

$$\int \frac{e^{(2x-1)/x}}{x^2} dx$$
$$= \int \frac{e^{2-1/x}}{x^2} dx$$

$$= \int e^u du \quad u = 2 - 1/x, \quad du = \frac{1}{x^2} dx$$

$$= e^u + C$$

$$= e^{2-1/x} + C$$

READY,  
GET SET,...

2:30

## INTEGRAL # 8

Z

$$\frac{e^x}{e^x + 1} dx$$





READY,  
GET SET,...

2:30

Z

$$\int \tan x \ln \cos x \, dx$$

## INTEGRAL # 9

Z

$$\int \tan x \ln \cos x \, dx$$

$$= \int u \, du \quad u = \ln \cos x, \quad du = \frac{1}{\cos x} (-\sin x) \, dx = -\tan x \, dx$$

$$= \frac{u^2}{2} + C$$

$$= \frac{1}{2} (\ln \cos x)^2 + C$$

READY,  
GET SET,...

2:30

Z

$$\int \tan x \sec^4 x \sqrt{1 + \sec^4 x} dx$$

## INTEGRAL # 10

Z

$$\int \tan x \sec^4 x \sqrt{1 + \sec^4 x} dx$$

$$= \frac{1}{4} \int u^{-1/2} du \quad u = 1 + \sec^4 x, \quad du = 4 \sec^3 x (\sec x \tan x) dx$$

$$= \frac{1}{6} u^{3/2} + C$$

$$= \frac{1}{6} (1 + \sec^4 x)^{3/2} + C$$

READY,  
GET SET,...

2:30



Z

$$\cos^5 x \sin 2x \, dx$$

## INTEGRAL # 11

Z

$$\cos^5 x \sin 2x \, dx$$

Z

$$= \cos^5 x (2 \sin x \cos x) \, dx$$

Z

$$= 2 \cos^6 x \sin x \, dx \quad u = \cos x, \quad du = -\sin x \, dx$$

Z

$$= 2 \int u^6 \, du$$

$$= \frac{2}{7} u^7 + C = \frac{2}{7} \cos^7 x + C$$



INTEGRAL # 12

$$\int \frac{2x^2 + 7x + 4}{x + 3} dx$$



INTEGRAL # 13

READY,  
GET SET,...

2:30







READY,  
GET SET,...

2:30

INTEGRAL # 14

Z

$$\int x \csc^2 x \, dx$$



Z

$$x \csc^2 x \, dx$$

Z

$$= x \cot x + \cot x \, dx$$

Integration by parts:

$$\begin{aligned}
 u &= x & dv &= \csc^2 x \, dx \\
 du &= dx & v &= -\cot x
 \end{aligned}$$

READY,  
GET SET,...

2:30

Z

$$\int \frac{x}{x^3 + 3x^2 + 3x + 1} dx$$

## INTEGRAL # 15

 $\int$ 

$$\frac{x}{x^3 + 3x^2 + 3x + 1} dx$$

 $\int$ 

$$= \frac{x}{x}$$